Algebra II

8-5

Remainder and Factor Theorems

Remainder Theorem -

When dividing $\frac{P(x)}{x - a}$, the remainder will always equal $P(a)$.

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Use synthetic substitution to find $P(c)$ for the given polynomial for the given c.

1) $P(x) = x^3 - 2x^2 - 5x - 7$; $c = 4$

	1	-2	-5	-7
4		4	8	12
	1	2	3	$\boxed{5}$

$P(4) = 4^3 - 2(4)^2 - 5(4) - 7$

$= 64 - 32 - 20 - 7 = \boxed{5}$

Note, both methods give the same result. Choose whichever method you prefer.

Factor Theorem -

When dividing $\frac{P(x)}{x - a}$, if the remainder is equal to zero, then $x - a$ is a factor of $P(x)$, and $\{a\}$ is a root of $P(x)$.

synonyms of *root*

↳ solution, x-intercept, or zero

Use the factor theorem to determine whether the binomial is a factor of $P(x)$.

9) $x + 1$; $P(x) = x^7 - x^5 + x^3 - x$

	1	0	-1	0	1	0	-1	0
-1		-1	1	0	0	-1	1	0
	1	-1	0	0	1	-1	0	0

Yes, x+1 is a factor

A root of the equation is given. Solve the equation.

17) $x^3 + 3x^2 - 3x - 9 = 0$; -3 $\{-3, \pm\sqrt{3}\}$

	1	3	-3	-9
-3		-3	0	9
	1	0	-3	0

$x^2 - 3 = 0$ Quadratic

$\sqrt{x^2} = \sqrt{3}$

$|x| = \sqrt{3}$

$x = \pm\sqrt{3}$

Find a polynomial equation with integral coefficients that has the given roots.

21) { 1, 2, -3 }

$(x-1)(x-2)(x+3) = 0$

$(x^2 - 2x - 1x + 2)(x+3) = 0$

$(x^2 - 3x + 2)(x+3) = 0$

$x^3 \cancel{-3x^2} + 2x + \cancel{3x^2} - 9x + 6 = 0$

$\boxed{x^3 - 7x + 6 = 0}$

$x - 1 = 0$ $x - 2 = 0$ $x + 3 = 0$

$x = 1$, $x = 2$, $x = -3$

$\{1, 2, -3\}$

Solve each equation given the two indicated roots.

29) $x^4 - 3x^3 - 8x^2 + 12x + 16$ { -1, 4, ± 2 }

	~~1~~	~~-3~~	~~-8~~	~~12~~	~~16~~
-1		~~-1~~	~~4~~	~~4~~	~~-16~~
	$1x^3$	$-4x^2$	$-4x$	16	0 ✓
4		4	0	-16	
	$1x^2$	$0x$	-4	0 ✓	

Each time you divide out a solution (zero remainder), use the new set of numbers for the next line of the division. I lightly cross out the numbers no longer needed.

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

± 2

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2-32even